

# *COBE* constraints on Kaluza–Klein cosmologies

V. Faraoni, F. I. Cooperstock and J. M. Overduin

*Department of Physics and Astronomy, University of Victoria  
P.O. Box 3055, Victoria, B.C. V8W 3P6 (Canada)*

## **Abstract**

A class of Kaluza–Klein cosmologies recently proposed is compared with observations of the cosmic microwave background. Some models are ruled out, while others turn out to be viable, with the number of extra dimensions being constrained by the *COBE* and Tenerife experiments.

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Kaluza–Klein (KK) theory [1] has been revived in recent years in the forms of supergravity [2], superstring theories [3], and theories with an extended gravitational sector. Some of the early ideas of Kaluza and Klein, like the presence of compact extra dimensions and the geometrical origin of scalar and gauge fields survive in these theories. Many of the papers currently published in the context of KK theories deal with cosmology.

In this Letter we discuss a class of KK cosmological models proposed recently by Cho ([4, 5] and references therein) and Cho and Yoon [6] (see also [7]). These models have not been studied in sufficient detail to decide if they are realistic. In particular, no prediction was given which, in conjunction with observations, makes these models falsifiable. Here we fill this gap for simple versions of these KK cosmologies. These models are seen as a possible alternative to ordinary inflation in [4], because they may solve some of the problems of the standard big bang model (including the horizon problem). In this sense the term “generalized inflation” is used in [4]. In the following we use the word inflation in the conventional sense. More precisely, this term denotes a regime in which the scale factor of the (4-dimensional) spacetime  $a(t)$  satisfies  $\ddot{a} > 0$  (a dot denoting differentiation with respect to the comoving time  $t$ ). Currently, a successful mechanism for the generation of density perturbations is available only in inflationary models of the universe [8], and we discard the non-inflationary models in [4]–[6] as non-viable. Should a non-inflationary mechanism for the generation of density perturbations turn out to be viable, our procedure of ruling out some models on the basis of their being non-inflationary would have to be reexamined. However, until this happens, we adopt the conventional idea that the generation of structures in the universe requires inflation [8].

The conclusions that we reach on the viability of the cosmologies of [4]–[6] depend on the details of these models and, strictly speaking, cannot be generalized. However, the cosmologies in [4]–[6] are representative of many models that appeared in recent literature on KK cosmology [9].

As a starting point, we consider the models of [4, 5]. The  $(4 + d)$ -dimensional spacetime manifold is assumed to be the product  $M \otimes K$ , where  $M$  is 4-dimensional and  $K$  is a  $d$ -dimensional submanifold ( $d \geq 1$ ). The  $(4 + d)$ -dimensional metric is given by

$$(\hat{g}_{AB}) = \begin{pmatrix} \hat{g}_{\mu\nu} & 0 \\ 0 & \hat{\phi}_{ab} \end{pmatrix}, \quad (1)$$

where<sup>1</sup>  $A, B, \dots = 0, 1, \dots, (4 + d)$ ,  $\mu, \nu, \dots = 0, 1, 2, 3$ , and  $a, b, \dots = 4, 5, \dots, (4 + d)$ .

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<sup>1</sup>For ease of comparison with the literature on inflation, we use units in which the speed of light and the reduced Planck constant assume the value of unity. The Planck mass is related to the Newton

It is assumed that  $\hat{g}_{\mu\nu}$  (called the “Jordan metric” in [5]) is a Friedmann–Lemaître–Robertson–Walker (FLRW) metric on  $M$  and  $\hat{\phi}_{ab}$  is a Riemannian metric on  $K$ , with the extra dimensions being spacelike. For simplicity it is assumed here that no gauge field corresponding to “off-diagonal” terms in the metric (1) is present and that the torsion tensor vanishes<sup>2</sup>. These simplifications are widely used in KK cosmology (e.g. [13]). In addition, the stress–energy tensor of matter in  $(4 + d)$  dimensions is assumed to vanish identically, because we are interested in the density perturbations originating during the inflationary era of the universe, which is believed to be dominated by a scalar field (of geometrical origin in KK theory). During this era, the other forms of matter can be neglected. Following [4], we introduce the quantities  $\phi$  and  $\rho_{ab}$  defined by

$$\phi = \left| \det(\hat{\phi}_{ab}) \right| , \quad (2)$$

$$\hat{\phi}_{ab} = \phi^{1/d} \rho_{ab} , \quad (3)$$

where  $|\det(\rho_{ab})| = 1$ . The theory is essentially vacuum general relativity in  $(4 + d)$  dimensions and is described by the Lagrangian density [4]

$$\mathcal{L}^{(4+d)} = - \frac{m_{pl}^2}{16\pi} \left( \hat{R} + \Lambda \right) \sqrt{-\det(g_{AB})} , \quad (4)$$

where  $\hat{\nabla}_\mu$  and  $\hat{R}$  are, respectively, the covariant derivative operator and the Ricci curvature of the metric  $\hat{g}_{AB}$ , and  $\Lambda$  is the cosmological constant of the  $(4 + d)$ –dimensional spacetime.

Dimensional reduction and the conformal transformation<sup>3</sup>

$$g_{\mu\nu} = \sqrt{\phi} \hat{g}_{\mu\nu} \quad (5)$$

together with the redefinition of the scalar field

$$\sigma = \frac{1}{2} \left( \frac{d+2}{d} \right)^{1/2} \ln \phi \quad (6)$$

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constant  $G$  by  $m_{pl} = G^{-1/2}$ . These units differ from those used in refs. [4]–[6]. Apart from this, and other minor differences in the symbols, our notations coincide with those of [4]–[6].

<sup>2</sup>The last assumption will be relaxed in the following.

<sup>3</sup>See [14] for a proof of the necessity and uniqueness of the transformation (5), (6). This transformation is widely used in many theories unifying gravity with the other forces.

lead to the “Einstein frame”, in which the theory is described by the Lagrangian density [4, 5]

$$\mathcal{L}' = -\frac{m_{pl}^2}{16\pi} \left[ R + \frac{1}{2} \nabla_\mu \sigma \nabla^\mu \sigma + V(\sigma) + \lambda (|\det(\rho_{ab})| - 1) \right] \sqrt{-g} , \quad (7)$$

where  $\nabla_\mu$  and  $R$  are, respectively, the covariant derivative and the Ricci curvature of the FLRW metric  $g_{\mu\nu}$  (designated the “Pauli metric” in [5]),  $g \equiv \det(g_{\mu\nu})$  and [4, 5]

$$V(\sigma) = R_K \exp(-\alpha\sigma) + \Lambda \exp(-\beta\sigma) , \quad (8)$$

$$\alpha = \beta^{-1} = \left( \frac{d+2}{d} \right)^{1/2} . \quad (9)$$

Here  $R_K$  is the Ricci curvature of the metric  $\rho_{ab}$  on the submanifold  $K$  and  $\lambda$  is a Lagrange multiplier introduced to obtain the constraint equation  $\det(\rho_{ab}) = 1$ . For simplicity, we omit from the Lagrangian density (7) the gauge fields included in [4]–[6]. This is justified by the fact that in the proposed cosmologies, the universe is dominated by the dilaton [4].

In [4] the theory described by the Lagrangian density (7) is interpreted as a 4-dimensional relativistic cosmology, where the dominant material source is the massless, minimally coupled, scalar field  $\sigma$  self-interacting via the potential (8), and it is suggested that this theory may describe an inflationary universe. However, for this interpretation it is necessary to consider the renormalized field

$$\bar{\sigma} = \frac{\sigma}{\sqrt{16\pi G}} \quad (10)$$

instead of  $\sigma$ . This point is crucial for our results because the arguments of the exponentials in the scalar field potential are changed by this renormalization. In fact, a failure to impose this renormalization would lead to incorrect conclusions regarding the compatibility of these cosmologies with observations.

We also introduce the quantities

$$\bar{R}_K = \frac{R_K}{16\pi G} , \quad \bar{\Lambda} = \frac{\Lambda}{16\pi G} , \quad \bar{\lambda} = \frac{\lambda}{16\pi G} . \quad (11)$$

The Lagrangian density of the cosmological theory with the correct coupling of the scalar field is then given by

$$\mathcal{L}' = - \left[ \frac{m_{pl}^2}{16\pi} R + \frac{1}{2} \nabla_\mu \bar{\sigma} \nabla^\mu \bar{\sigma} + W(\bar{\sigma}) + \bar{\lambda} (|\det(\rho_{ab})| - 1) \right] \sqrt{-g} , \quad (12)$$

where

$$W(\bar{\sigma}) = \bar{R}_K \exp\left(-\sqrt{16\pi} \alpha \frac{\bar{\sigma}}{m_{pl}}\right) + \bar{\Lambda} \exp\left(-\sqrt{16\pi} \beta \frac{\bar{\sigma}}{m_{pl}}\right). \quad (13)$$

It is to be noted that the cosmological constant  $\Lambda$  has been absorbed in the potential of the new scalar field as a result of the conformal transformation and hence there is no cosmological constant in the 4-dimensional spacetime. The Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \quad (14)$$

combined with the expressions for the energy density and pressure of the scalar field

$$\rho = \frac{\dot{\bar{\sigma}}^2}{2} + W, \quad P = \frac{\dot{\bar{\sigma}}^2}{2} - W \quad (15)$$

gives

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3}(\dot{\bar{\sigma}}^2 - W). \quad (16)$$

As is clear from eq. (16), a necessary condition for inflation is  $W > 0$ . We now consider various possible scenarios, corresponding to different values of  $R_K$ ,  $\Lambda$  and  $d$ , which act as parameters of the theory. The possibilities that  $R_K$  is positive, zero, or negative are considered in [4], and we will not discard any of these *a priori*. Different choices are motivated as follows. A compact internal space  $K$  corresponds to  $R_K < 0$  in these notations [4]. For example, the case in which  $K$  is a  $d$ -dimensional sphere has been considered in the literature [2, 20]. Various KK models in which the submanifold  $K$  is Ricci-flat have also been considered (e.g. [24]), and two in particular are worth mentioning. One occurs if there is only one extra dimension ( $d = 1$ ); the other is the case in which the compactification of the  $K$  submanifold is performed by the points identification  $x^a \rightarrow x^a + 2\pi$ , in the spirit of Kaluza's original paper. We have the following results:

**Case 1)**  $W = W(\bar{\sigma})$

- **Case 1a)**  $\bar{R}_K = 0$ ;  $\bar{\Lambda} \leq 0$ :  $W \leq 0$  and inflation does not occur.
- **Case 1b)**  $\bar{R}_K = 0$ ;  $\bar{\Lambda} > 0$ : the potential (13) reduces to a single exponential. This kind of potential has been studied in the context of power-law inflation ([15, 16]—see also [11]). To be specific, the scalar field potential

$$U(\bar{\sigma}) = U_0 \exp\left(-\sqrt{\frac{16\pi}{p}} \frac{\bar{\sigma}}{m_{pl}}\right), \quad (17)$$

where  $V_0$  and  $p$  are constants, is required in order to have a scale factor  $a(t) \propto t^p$  (which describes inflation if  $p > 1$ ) in the FLRW metric  $g_{\mu\nu}$ . The power law inflationary scenario can be solved exactly, and the density perturbations are described by a power law spectrum with index  $n - 1$ , where  $n = 1 - 2/p$  (see [11] for a review). By comparing eqs. (13), (9) in the case  $\bar{R}_K = 0$  with eq. (17), one obtains immediately

$$n = 1 - \frac{2d}{d+2} \leq \frac{1}{3}. \quad (18)$$

The  $1\sigma$  results of the *COBE* [18] experiment provide us with the limit  $n = 1.1 \pm 0.5$ . The combined statistical analysis of the Tenerife and *COBE* experiments [19] shows that, taking  $1\sigma$  limits from both experiments, they are consistent if  $n \geq 0.9$ . Therefore, the cosmological model under consideration is ruled out.

- **Case 1c)**  $\bar{R}_K < 0$ ;  $\bar{\Lambda} \leq 0$ :  $W \leq 0$  and inflation does not occur.
- **Case 1d)**  $\bar{R}_K < 0$ ;  $\bar{\Lambda} > 0$ : this case can be reduced to the double exponential potential studied recently by Easter [22]. The corresponding inflation can mimic most of the usual models of inflation. The parameters in [22] assume the values

$$\mathcal{A} = \bar{\Lambda}, \quad \mathcal{B} = -\bar{R}_K, \quad \xi = \sqrt{2}\beta, \quad m = \frac{d+2}{d}. \quad (19)$$

Easter studies the constraint imposed by the observations of microwave background anisotropies and concludes that if  $\xi \geq 0.5$  no values of  $m$  and  $\xi$  produce a viable perturbation spectrum. In the present case,  $d \geq 1$  implies  $\xi \geq (2/3)^{1/2} \simeq 0.82$ . Therefore this scenario is ruled out.

- **Case 1e)**  $\bar{R}_K > 0$ ;  $\bar{\Lambda} = 0$ : we compare eqs. (13), (9) and (17) to obtain  $p = d/(d+2) < 1$ , which does not correspond to an inflationary scenario.
- **Case 1f)**  $\bar{R}_K > 0$ ;  $\bar{\Lambda} > 0$ : this situation is not covered in<sup>4</sup> [22] (due to the restriction  $m > 1$  in that paper). It is unclear if this model is inflationary, the answer depending on the relative amplitudes of  $\bar{R}_K$ ,  $\bar{\Lambda}$  and the value of  $d$ . In any case, the potential (13) is not a single exponential, which excludes the power law inflation [16, 21] expected in KK theories [10]. A detailed study of models (unperturbed, or including density perturbations) with both  $\bar{R}_K$  and  $\bar{\Lambda}$  positive

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<sup>4</sup>A double exponential potential is also considered in [23], but the range of the parameters considered there does not include case 1f).

has not been given, and without a specification of the values of  $\bar{R}_K$ ,  $\bar{\Lambda}$ ,  $d$ , the comparison with observations appears problematic, to say the least. In the absence of a clear prescription for a meaningful inflationary model, the most general case  $\bar{R}_K, \bar{\Lambda} > 0$  will not be discussed here. However, we can test the model in a meaningful approximation. A high value of  $d$  ( $d \geq 36$ – $40$ ) has been advocated by many authors in order to generate a large amount of entropy in the 4-dimensional universe [12]. By approximating eq. (13) for  $d \gg 1$ , we obtain the exponential potential  $W_0 \exp(-\sqrt{16\pi}\bar{\sigma}/m_{pl})$ , where  $W_0 = \bar{R}_K + \bar{\Lambda}$ . The potential has the power law form (17) with  $p = 1$ , which corresponds to a coasting universe (scale factor linear in time) and is not inflationary. This conclusion applies to all models with  $\bar{R}_K + \bar{\Lambda} > 0$  if the number of extra dimensions is high.

- **Case 1g)**  $\bar{R}_K > 0$ ;  $\bar{\Lambda} < 0$ : the same conclusion as in case 1f).

So far, no explicitly inflationary solutions consistent with observations have been found. We consider now the cosmologies introduced in [6]. In these models the starting point is Einstein–Cartan theory in  $(4+d)$  dimensions, with a non-vanishing torsion tensor. The effective theory in 4 dimensions is similar to the models in [4, 5]. The cosmologies in [6] include two scalar fields  $\sigma$  and  $\varphi$ . The previous remark on the interpretation of this theory as 4-dimensional relativistic cosmology dominated by scalar fields applies again: it is possible to consider the usual Einstein equations and a cosmological scenario only after the scalar fields are redefined according to eq. (10) and to the analogous  $\bar{\varphi} = \varphi/\sqrt{16\pi G}$ . The potential  $V(\sigma, \varphi)$  in eq. (5.2) of ref. [6] has to be corrected accordingly. The particular cases given by eqs. (5.6) and (5.7) in [6] appear to be particularly interesting, since it is claimed that they may give rise to inflation. In the former situation, the correct potential is

$$W(\bar{\sigma}, 0) = \frac{m_{pl}^2}{16\pi} \left[ -\frac{Ad}{4} \exp\left(-\sqrt{\frac{16\pi(d+2)}{d}} \frac{\bar{\sigma}}{m_{pl}}\right) - \Lambda \exp\left(-\sqrt{\frac{16\pi d}{d+2}} \frac{\bar{\sigma}}{m_{pl}}\right) \right], \quad (20)$$

where the constant  $A$  is defined in [6]. We can draw some conclusions immediately.

**Case2:**  $W = W(\bar{\sigma}, 0)$

- **Case 2a)**  $A > 0$ ;  $\Lambda \geq 0$ :  $W < 0$  and there is no inflation.
- **Case 2b)**  $A > 0$ ;  $\Lambda < 0$ : this corresponds again to the case considered in [22] with parameters

$$\mathcal{A} = \bar{\Lambda}, \quad \mathcal{B} = \frac{Ad}{4} \frac{m_{pl}^2}{16\pi}, \quad \xi = \left(\frac{2d}{d+2}\right)^{1/2} \geq 0.82, \quad m = \frac{d+2}{d}. \quad (21)$$

The situation is the same as in case 1d) and this scenario is excluded as well by the microwave background experiments.

- **Case 2c)**  $A = 0$ ;  $\Lambda \geq 0$ :  $W \leq 0$  and inflation does not occur.
- **Case 2d)**  $A = 0$ ;  $\Lambda < 0$ : the potential reduces to the form (17) and there is power law inflation with  $p = (d+2)/d$ . This scenario is ruled out by *COBE* analogously to case 1b).
- **Case 2e)**  $A < 0$ ;  $\Lambda = 0$ : the potential reduces to the form (17) with  $p = d/(d+2)$ , which is not inflationary.
- **Case 2f)**  $A < 0$ ;  $\Lambda \neq 0$ : the same conclusion as in case 1f), but now  $W_0 = (64\pi)^{-1} |A| dm_{pl}^2 - \bar{\Lambda}$ .

Again, no explicitly inflationary universes consistent with observations have been obtained in case 2).

If, instead, the potential given in eq. (5.7) of [6] (corrected after the rescaling of  $\varphi$ ) is adopted, one has

$$W(0, \bar{\varphi}) = \frac{m_{pl}^2}{16\pi} \left\{ \frac{A}{4} \left[ \exp \left( -\sqrt{\frac{32\pi}{d(d-1)}} (d+1) \frac{\bar{\varphi}}{m_{pl}} \right) - (d+1) \exp \left( -\sqrt{\frac{32\pi}{d(d-1)}} \frac{\bar{\varphi}}{m_{pl}} \right) \right] - \Lambda \right\}. \quad (22)$$

In this case the effective cosmological constant  $\Lambda_4 = -\bar{\Lambda}$  is induced in the 4-dimensional spacetime.

**Case 3:**  $W = W(0, \bar{\varphi})$

- **Case 3a)**  $\Lambda = 0$ ;  $A = 0$ :  $W = 0$  and there is no inflation.
- **Case 3b)**  $W(0, \bar{\varphi})$ ;  $\Lambda = 0$ ;  $A > 0$ : the same conclusion holds here as in case 1f), but no conclusion can be reached for  $d \gg 1$ .
- **Case 3c)**  $W(0, \bar{\varphi})$ ;  $\Lambda = 0$ ;  $A < 0$ : this case is reduced to the scenario studied in [22] with the values of the parameters

$$\mathcal{A} = |A|(d+1) \frac{m_{pl}^2}{64\pi}, \quad \mathcal{B} = |A| \frac{m_{pl}^2}{64\pi}, \quad \xi = \frac{2}{\sqrt{d(d-1)}}, \quad m = d+1. \quad (23)$$



Easter [22] concludes that if  $m\xi^2 \leq 0.15$  (with  $m > 1$ ) the resulting spectrum satisfies the *COBE* and *QDOT* (with cold dark matter) constraints for all values of  $\bar{\varphi}$ . On the contrary, the perturbation spectrum is incompatible with these observations if  $\xi \geq 0.5$ . The former inequality is equivalent to  $d \geq 29$ , and the latter is equivalent to  $d \leq 4$ . Therefore the model is not viable if  $d \leq 4$  and is viable if  $d \geq 29$ .

The analysis of the cases  $d = 5, \dots, 28$  requires the expression of the spectral index of density perturbations in this potential, as computed in [22]:

$$n = 1 + \frac{m\xi^2}{(m-1+y)^2} \left[ 2(m-1)^2(y-1) - my^2 \right], \quad (24)$$

where the variable  $y$  is related to the scalar field by

$$1 - y = \exp \left( -\sqrt{\frac{32\pi d}{d-1}} \frac{\bar{\varphi}}{m_{pl}} \right). \quad (25)$$

In the relevant range of  $y \in [0, 1)$  [22],  $n$  is a monotonically increasing function of  $d$ , for all  $d \geq 4$ . The constraint  $n \geq 0.9$  [19] is satisfied if  $y \geq y_0$ . The root  $y_0$  of the equation  $n(y) = 0.9$  is less than unity only if  $d \geq 7$  and  $\bar{\varphi} \geq \bar{\varphi}_0$ , where  $\bar{\varphi}_0$  is a decreasing function of  $d$ . Therefore, the model is viable only if  $d \geq 7$ , and only over a range of values of the scalar field  $\bar{\varphi}$ . To summarize, this model cannot be viable for  $d < 7$ . It is viable (for some values of the scalar field  $\bar{\varphi}$ ) over the range  $7 \leq d \leq 29$  and, for  $d \geq 29$ , it is viable in all cases.

It is remarkable that observations of the microwave background constrain the number of extra dimensions in a higher dimensional theory. This possibility is unique, since higher dimensional theories [2, 3] are usually so complicated that the dimension cannot be varied as a parameter. It is hoped that the KK models considered here serve as a toy model for more fundamental theories, and that the restrictions on the dimensionality of spacetime has a broader range of validity. Cosmological perturbations in KK theories have been considered in the literature [17], but the possibility of relating the number of extra dimensions to observable quantities has apparently gone unnoticed until now.

- **Case 3d)**  $\Lambda > 0$ ;  $A = 0$ :  $W < 0$  and there is no inflation.

- **Case 3e)**  $\Lambda \neq 0$ ;  $A \neq 0$ : the same as case 1f). However, in the approximation  $d \gg 1$  the potential (22) reduces to

$$W(0, \bar{\varphi}) \approx \frac{m_{pl}^2}{16\pi} \left[ \frac{A}{4} \exp \left( -\sqrt{32\pi} \frac{\bar{\varphi}}{m_{pl}} \right) - \left( \Lambda + \frac{Ad}{4} \right) \right]. \quad (26)$$

The cosmological constant  $\Lambda_4 = - \left[ (64\pi)^{-1} Adm_{pl}^2 + \bar{\Lambda} \right]$  is induced in the 4-dimensional spacetime. If  $\Lambda = -Ad/4$ , there is no cosmological constant in the 4-dimensional spacetime. Then, if  $A < 0$ ,  $W < 0$  and there is no inflation. If instead  $A > 0$ , the scale factor has the form  $a(t) \propto \sqrt{t}$  describing a radiation-dominated universe.

- **Case 3f)**  $\Lambda < 0$ ;  $A = 0$ : the potential corresponds to a de Sitter expansion.

Our results can be summarized as follows. We adopt the point of view that inflation is a necessary ingredient for a successful cosmological model. Then the cosmological models of cases 1a), 1c), 1e) 1f) and 1g) for  $d \gg 1$ , 2e), 2c), 2e), 3a), 3d), 3e) if  $d \gg 1$  and  $\Lambda_4 = 0$  are ruled out because inflation does not occur. The models of cases 1b), 1d), 2b), 2d), and 3c) if  $d \leq 6$  are incompatible with observations of the cosmic microwave background anisotropies. In general, the cases 1f), 1g), 2f), 3b) and 3e) require a more detailed study that is beyond the focus of this work. The same applies to the model of [7], which is inflationary. Case 3c) is viable if  $d \geq 7$ . In this case, the *COBE* and Tenerife experiments constrain the number of extra dimensions.

It is to be noted that if the renormalization (10), (11) is missed, and the incorrect potential given by eqs. (8), (9) is used, one would obtain the erroneous result that many of the models in [4] give rise to power law inflation, with spectral indices of perturbations compatible with the *COBE* and Tenerife experiments.

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